

The structure of weighted densities

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Density is one of the possibilities to measure how large a subset of the set of positive integers is. The best known type of densities are weighted densities.

Let $f : \mathbb{N} \rightarrow (0, \infty)$ be a weight function such that the conditions

$$\sum_{n=1}^{\infty} f(n) = \infty,$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{\sum_{i=1}^n f(i)} = 0$$

are satisfied.

For $A \subset \mathbb{N}$ and $n \in \mathbb{N}$ denote $A_f(n) = \sum_{a \in A, a \leq n} f(a)$ and define

$$\underline{d}_f(A) = \liminf_{n \rightarrow \infty} \frac{A_f(n)}{\mathbb{N}_f(n)} \quad \bar{d}_f(A) = \limsup_{n \rightarrow \infty} \frac{A_f(n)}{\mathbb{N}_f(n)}$$

the lower and upper f -densities of A , respectively.

We present relations between weighted densities determined by several weight functions.