On the minimality of certain de la Vallée Poussin type projections

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Let us denote by $C_0(2\pi)$ the space of continuous 2π-periodic functions equipped with the maximum norm. Denoting the subspace of trigonometric polynomials of degree at most n by $\mathcal{T}_n$, the Fourier projection $F_n : C_0(2\pi) \to \mathcal{T}_n$ (for which $F_n f$, $(f \in C_0(2\pi))$ is the n-th partial sum of the trigonometric Fourier series of f) is a linear operator with the property $F_n|_{\mathcal{T}_n} \equiv$id. In fact, the Faber–Marcinkiewicz–Berman theorem states that F_n has the minimal norm among all projections from $C_0(2\pi)$ to $\mathcal{T}_n$ with the above property. This result stays true if we replace $C_0(2\pi)$ with the space of (Lebesgue) integrable functions $L_1(0, 2\pi)$. In this talk, for natural numbers $m \leq n$, we consider the sets of generalised projections $\{P \in \mathcal{L}(X, \mathcal{T}_n) : P|_{\mathcal{T}_m} \equiv \text{id}\}$, where $X = C_0(2\pi)$ or $X = L_1(0, 2\pi)$. Note that for $m := n$, we obtain the previously described operators. It turns out that in many cases, the minimal element of these sets are obtained by taking the arithmetic means of some Fourier projections, i.e. it is some kind of (generalized) de la Vallée Poussin sum. Our aim is to describe the occurrences of these cases in detail, giving necessary and sufficient conditions for the minimality.

References

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