Convergence of integral operators in variable Lebesgue spaces

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The classical Hardy-Littlewood maximal operator is bounded not only on the classical Lebesgue spaces $L_p(\mathbb{R}^d)$ (in case of $p > 1$), but (in case of $p_- > 1$ and $1/p(\cdot)$ is log-Hölder continuous) on the variable Lebesgue spaces $L_{p(\cdot)}(\mathbb{R}^d)$, too. These results have been generalized for the maximal operator $M_r^{\gamma,\delta}$ with the help of generalized Φ-functions in [1]. We have investigated in [2] a general multi-dimensional integral operator V_T. Under the condition that the kernel function of V_T is in a suitable Herz space, we get several convergence theorems about norm and almost everywhere convergence and convergence at Lebesgue points. The multi-dimensional convergence is investigated over cones and cone-like sets. As special cases we consider three multi-dimensional integral operators, the θ-summation of Fourier transforms and Fourier series and the discrete wavelet transforms. The convergence results are formulated for functions from the Wiener amalgam spaces and variable Lebesgue spaces, too.

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