Reducing graphs while preserving the number of their perfect matchings

Miklós Bartha[1] and Miklós Krész[2]

[1]Department of Computer Science
Memorial University of Newfoundland, St. John's, Canada
[2]Juhász Gyula Faculty of Education
University of Szeged, Hungary

bartha@mun.ca, kresz@jgypk.szte.hu

In a recent paper, Levit and Mandrescu [**?**] gave an interesting characterization of graphs having a unique perfect matching and satisfying the König property at the same time. These graphs are exactly the ones that can be reduced to the empty graph by iteratively deleting a leaf vertex together with the vertex adjacent to it. The underlying leaf elimination procedure is straightforward to implement, so that the property of having a unique perfect matching is decidable in linear time for graphs obeying the König property. This observation is in line with our long-standing conjecture that the property of having a unique perfect matching is decidable for all graphs in linear time.

In this paper we add a second component, called line reduction, to the reduction procedure defined by leaf elimination. Line reduction, which has been studied in [**?**], also preserves the number of perfect matchings and has a linear-time implementation. It is therefore natural to combine leaf elimination and line reduction, and see if the resulting reduction is still confluent and can be implemented in linear time. We give a positive answer to both of these questions. The algorithm used in the implementation is based on the well-known union-find technique [**?**] to implement the dynamic union of disjoint sets. The result shows that our conjecture is true for a much larger class of graphs having a unique perfect matching, namely the ones that can be reduced to the empty graph by the combined leaf-and-line reduction. Such graphs are called *zero-reducible.*

Interestingly, line reduction, too, preserves the König property in graphs. The inverse of line reduction, however, does not. Nevertheless, we investigate the possibility whether, on the analogy of the main result of [**?**], one could come up with a characterization of zero-reducible graphs in terms of the König property. Unfortunately it turns out that such a characterization is not likely to exist, and the problem of finding the independence number of zero-reducible graphs is still NP-complete.

References

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