Detecting negative cycles with Tarjan's breadth-first scanning algorithm

Tibor Ásványi

Eötvös Loránd University, Faculty of Informatics
Budapest, Hungary

`asvanyi@inf.elte.hu`

The Bellman-Ford algorithm [1, 2] solves the single-source shortest path problem on directed, weighted graphs in the general case: Edges with negative weights are allowed, provided that there is no negative cycle reachable from the source vertex s (otherwise there is no shortest path to some vertexes reachable from s). The algorithm runs in $\Theta(nm)$ time, where n and m are the numbers of vertexes and edges respectively. Therefore many improvements have been suggested.

One of them is Tarjan's breadth-first scanning (BFscan) algorithm [3, 4] (also known as Queue-based Bellman-Ford algorithm [5]). It runs in $O(nm)$ time, which may not seem a great improvement, but BFscan performs much better than the Bellman-Ford algorithm in practice.

The original BFscan terminates, *iff* there is no negative cycle reachable from the source vertex s. Otherwise the method never halts [4]. In order to make BFscan robust, a few distinct supplements have been proposed [4, 6]. (They compute the shortest paths, *or* detect some negative cycle reachable from s.) In this paper we introduce a new completion, and compare it with the others known to us.

References

[1] BELLMAN, R.E., On a Routing Problem, *Quarterly of Applied Mathematics, 16 (1958), 87-90.*

[2] CORMEN, T.H., LEISERSON, C.E., RIVEST, R.L., STEIN, C., Introduction to Algorithms, *The MIT Press*, 2009.

[3] TARJAN, ROBERT ENDRE, A unified approach to path problems, *J. Assoc. Comput. Mach., 28, pp. 577-593*, 1981.

[4] TARJAN, ROBERT ENDRE, Data Structures and Network Algorithms, *CBMS-NSF Regional Conference Series in Applied Mathematics*, 1987.

[5] SEDGEWICK, R., WAYNE, K., Algorithms, 4th Edition *Addison-Wesley Professional*, 2011. ISBN 0-321-57351-X (Ebook: http://algs4.cs.princeton.edu/home/)

[6] WEISS, M.A., Data Structures and Algorithm Analysis, *Addison-Wesley*, 1995, 1997, 2007, 2012, 2013.